P425/1
PURE MATHEMATICS
Paper 1
August 2016
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B**.
- *Any additional question(s) answered will* **not** *be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

Turn Over

SECTION A (40 MARKS)

Answer **all** *questions in this section*

1. Solve the simultaneous equations $2x - y + 2z = 6$ and $\frac{x+2}{3} = \frac{y+2}{4} = \frac{z+2}{5}$ (5 marks)

2. Solve the equation $(1 - \sin x)^2 + (1 + \cos x)^2 = 1$ for $0° \leq x \leq 180°$. (5 marks)

3. The points A, B and C have position vectors $4\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}$, $6\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} + 10\mathbf{j} + 3\mathbf{k}$ respectively. If A, B and C are the vertices of a triangle show that angle ABC is a right angle.

   (5 marks)

4. (i) By eliminating ε from $y = \sin(\varepsilon + \omega t)$, form a differential equation. (3 marks)

   (ii) State the order of the differential equation in (i) above. (1 mark)

5. The distance of the point (2, -1) from the line $y = \frac{3}{4}x + p$ is twice its distance from the line $y = -\frac{3}{4}x$. Find the value of p. (4 marks)

6. Using the substitution of $u = \sin\theta$, evaluate $\int_0^{\pi} \sin^2\theta \cos^3\theta d\theta$ (6 marks)

7. Solve the equations $\log_b a + 2\log_a b = 3$ and $\log_9 a + \log_9 b = 3$. Given that $a \neq b$ (6 marks)

8. The radius of a sphere increases at a rate of $0.01 cms^{-1}$. Find the rate at which the (i) surface area increases,

   (ii) volume increases,
   when the radius is 21cm.

   (5 marks)

SECTION B (60 MARKS)

Answer any **five** questions from this section.

9. Express $\frac{2x^2 - 5x+7}{(4x^2 - 9)(x + 2)}$ in partial fractions. Hence expand $\frac{2x^2 - 5x + 7}{(4x^2 - 9)(x + 2)}$ in ascending powers of x up to the term containing x^2.

(12 marks)

10. Given the curve $y = \frac{(x - 2)^2}{x + 1}$,

(i) Determine the turning point of y.

(ii) Find the region where the curve is not defined.

(iii) Sketch the curve.

(12 marks)

11. (a) Given that the point C divides the line $\overline{AB}$ in the ratio 1:2 and the position vectors of A and C are $-4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + 12\mathbf{k}$ respectively, find the coordinates of point B.

(5 marks)

(b) A plane contains points A(4, -6, 5) and B(2, 0, 1). A perpendicular to the plane from the point P(0, 4, -7) intersects the plane at point C. Find the Cartesian equation of the line $\overline{PC}$.

(7 marks)

12. (a) Solve the equation $7\cot x - \operatorname{cosec} x = 5$. For $0 \leq x \leq \frac{3}{2}\pi$. (5 marks)

(b) Given triangle PQR, prove that $\tan\left(\frac{Q - R}{2}\right) = \frac{q - r}{q + r} Cot \frac{P}{2}$.

Hence solve the triangle with two sides 5cm and 7cm including angle 45°.

(7 marks)

13. (a) Use De Moivre's theorem to express $\tan 4\theta$ in terms of $\tan\theta$.

(5 marks)

(b) Solve the equation $z^3 + 80 = 0$ (7marks)

Turn Over

14. (a) Given that $x = \sin\theta$ and $y = 1 - \cos\theta$, show that
$$\left(\frac{d^2y}{dx^2}\right)^2 - \left(1 + \left(\frac{dy}{dx}\right)^2\right)^2 = 0$$
(5 marks)

(b) The displacement x of a particle at any time t is given by $x = \sin t$. Find the mean value of its velocity over the interval $0 \le t \le \frac{\pi}{2}$ with respect to;
(i) time,
(ii) displacement. (7 marks)

15. (a) If the line $2x - 3y + 26 = 0$ is a tangent to the circle $x^2 + y^2 - 4x + 6y - 104 = 0$, find the coordinates of the point of contact. (6 marks)

(b) Find the equation of the circle which passes through the points (1, 1) and (1, -1) and is orthogonal to $x^2 + y^2 = 4$.
(6 marks)

16. In a certain city, the rate at which buildings are collapsing is proportional to those that have already collapsed. If initially B_o buildings have already collapsed,

(a) Show that $B = B_0e^{kt}$ where k is a constant and B_o is the number of buildings that have already collapsed. (8 marks)

(b) If the number of collapsed buildings doubled the initial number in 10 years, find the value of k. (2 marks)

(c) Determine the number of buildings that would have collapsed after 30 years in terms of the initial number B_o. (2 marks)

END